King Fahd University of Petroleum and Minerals
College of Computer Sciences and Engineering
Information and Computer Science Department

ICS 253: Discrete Structures I
Summer semester 2017-2018
Major Exam #1, Saturday July 14, 2018
Time: **100** Minutes

Name: ____

ID#: ______________________

Instructions:
1. The exam consists of 8 pages, including this page, containing 7 questions.
2. Answer all 7 questions. *Show all the steps*.
3. Make sure your answers are **clear** and **readable**.
4. The exam is closed book and closed notes. No calculators or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket.
5. If there is no space on the front of the page, use the back of the page.

Question	Maximum Points	Earned Points
1	15	
2	10	
3	15	
4	25	
5	15	
6	10	
7	10	
Total	**100**	

Rules of Inference:

$p \to (p \lor q)$	Addition	$[\neg q \land (p \to q)] \to \neg p$	Modus Tollens
$(p \land q) \to p$	Simplification	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$[(p) \land (q)] \to (p \land q)$	Conjunction	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllogism
$[p \land (p \to q)] \to q$	Modus Ponens	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution
$\forall x P(x) \to P(a)$ for all a	Universal Instantiation	$\exists x P(x) \to P(a)$ for some a	Existential Instantiation
$P(a)$ for all a $\to \forall x P(x)$	Universal Generalization	$P(a)$ for some a $\to \exists x P(x)$	Existential Generalization

Q1: [15 points] Answer the following questions.

a) [8 points] Let p, q, and r be the propositions

p: Belgian players have a lot of practice.
q: The coach has an effective strategy.
r: The Belgian team wins the match.

Write these propositions using p and q and logical connectives (including negations).

i. Belgian players have a lot of practice, but the coach does not have an effective strategy.

ii. It is necessary and sufficient that the Belgian players have a lot of practice, for the Belgian team to win the match.

iii. Unless the coach has an effective strategy, the Belgian team would not win the match.

b) [7 points] Without using a truth table, prove that $[(p \vee q) \wedge \neg p] \rightarrow q$ is a tautology.

Q2: [10 points] Recall the island of the knights and knaves, where knights always tell the truth and knaves always lie. You meet three inhabitants: Alice, Rex and Bob. Determine, if possible, who is knight and who is knave, if they said the following: Alice says that Rex is a knave. Rex says that it's false that Bob is a knave. Bob claims, “I am a knight or Alice is a knight.” Make sure you clearly justify your answer.

Q3: [15 points] Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

Hint: Use truth tables and discard illogical combinations!

Q4: [25 points] Let $F(x, y)$ be the statement "x can fool y," and $K(z)$ be the statement "z is a KFUPM student." where the domain of the variables x, y, and z consists of all people in the world. Use quantifiers and predicates to express each of these statements, where no negation is outside a quantifier or an expression involving logical connectives. You are **not** allowed to use "$\exists!$".

a) [5 points] Ahmad, who is a student at KFUPM, cannot fool Salim.

b) [5 points] KFUPM students can fool Lionel.

c) [5 points] No one can fool any KFUPM student.

d) [5 points] There is a KFUPM student who was fooled by every other student at KFUPM.

e) [5 points] There is exactly one non-KFUPM student who can fool every KFUPM student.

Q5: [15 points] Answer the following questions.

a) (6 points) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. If it is false, prove it.

i. $\forall x\left(x \neq 0 \rightarrow \exists y(xy = 1)\right)$

ii. $\forall x \exists y(x = y^2)$

iii. $\exists y \forall x(x + y = 1)$

b) (5 points) Show that the hypotheses "It is not raining or Yvette has her umbrella," "Yvette does not have her umbrella or she does not get wet," and "It is raining or Yvette does not get wet" imply that "Yvette does not get wet."

c) (4 points) Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1.	$\forall x(P(x) \vee Q(x))$	Premise
2.	$P(c) \vee Q(c)$	Universal Instantiation from (1)
3.	$P(c)$	Simplification from (2)
4.	$\forall x P(x)$	Universal generalization from (3)
5.	$Q(c)$	Simplification from (2)
6.	$\forall x Q(x)$	Universal generalization from (5)
7.	$\forall x P(x) \vee \forall x Q(x)$	Conjunction from (4) and (6).

Q6: [10 points] Prove that if *m* and *n* are integers and *mn* is even, then *m* is even or *n* is even.

Q7: [10 points] Answer the following questions.

a) (2 points) List the elements of $\mathcal{P}(\{\Phi, \{a\}\})$ where $\mathcal{P}$ denotes the power set and Φ denotes the empty set.

b) (4 points) For $A_i = [-i, i]$, where i is a positive integer and $[a, b]$ represents all real numbers x such that $a \leq x \leq b$.

i. Find $\bigcup_{i=1}^{\infty} A_i$

ii. Find $\bigcap_{i=1}^{\infty} A_i$

c) (4 points) Draw the Venn diagram for $(A \cap \bar{B}) - C$ of the sets A, B and C.